Tiered Problems for a Variety of Middle School Math Topics

Grade 7

Grade 8

7th Grade Unit: Algebraic Expression and Integers
Lesson: Evaluating Expressions

Goal: Students evaluate variable expressions by substitution and solve word problems by evaluating expressions.

Green level problems:

1. Evaluate $3ab + \frac{c}{2}$ for $a = 2$, $b = 5$, and $c = 10$

2. Write an expression for the number of kilometers a dolphin travels in h hours swimming at 8 km/hr. Then find the number of kilometers the dolphin travels in 3 hours.

Blue Level Problems:

3. If a = 4.9 and b = $1\frac{5}{9}$, determine the value of the reciprocal of b/a.

Express your answer as a common fraction.

4. The cost of rope at hardware store depends on its length in feet and its price per foot. Write a variable expression using 4 variables to represent the cost of 2 ropes that have different lengths and price per foot. Explain what each variable represents. Use the expression to evaluate the total cost to buy a 28-foot rope at $.23 per foot and a 36-foot rope at $.39 per foot.

Black Level Problems:

5. Heron's formula (sometimes called the semi-perimeter formula) says that if a triangle has sides lengths a, b, and c, then the area of the triangle is given by $A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{(a+b+c)}{2}$. As a decimal to the nearest tenth, how many square inches are in the area of the triangle with sides of length 4, 5, and 6 inches?

6. **Shark Problem.** A shark takes 20 minutes to swim with the current from its cave to the feeding grounds. Later, it returns against the current, taking 30 minutes.

a. Define variables for the shark's speed through the water and for the speed of the current (the speed at which the water is moving). Use feet per minute (ft/min). Then write two expressions, one for the distance the sharks swims out and the other for the distance it swims back.

b. Evaluate the expression in part (a) if:

i. The shark swims 400 ft/min and the current is 100 ft/min;

ii. The shark swims 80 ft/min and the current is 50 ft/min.

Solutions are provided here for the Blue and Black problems:

Blue Solutions

3. The reciprocal of $\frac{b}{a}$ is $\frac{a}{b}$, so let's just find $\frac{a}{b}: \frac{4.9}{1\frac{5}{9}} = \frac{\frac{49}{10}}{\frac{14}{9}} = \frac{49}{10} x \frac{9}{14} = \frac{63}{20}$.

4. Sample answer : $p\ell + cf$, where p is the cost per foot, ℓ is the length in feet of one rope and c is the cost per foot and f is the length in feet of the other rope ; \$20.48

Black Solutions

5. First, we calculate s, which is the semi-perimeter. $s = \frac{4+5+6}{2} = 7.5$.

Using this value and the three side lengths in the formula, we get

$A = \sqrt{7.5(7.5-4)(7.5-5)(7.5-6)} = \sqrt{98.4375}$ ≈ 9.9 square inches.

6. a. 20 (x + y) = no. of ft out, 30 (x – y) = no. of ft back
   b. i. 10000 ft out, 9000 ft back
      ii. 2600 ft out, 900 ft back
   c. Shark swims 87.5 ft/min.

7th Grade Unit: Factors Fractions and Exponents

Lesson: Prime Factorization, Greatest Common Factor and Simplifying Fractions

- Goals: Students learn to find the prime factorization of a number, the greatest common factor (GCF) of two or more numbers, to find equivalent fractions and to write fractions in simplest form.

Green level problems:

1. Use prime factorization to find the GCF of $27x^2y^3, 46x^2y$

2. Write in simplest form: $\dfrac{x^2y}{3yz}$

Blue Level Problems:

3. What is the sum of the three distinct prime factors of 47,432?

4. In a certain code, each of the 26 letters of the English alphabet is represented by a number (A=1, B=2, C=3,... Z=26). A word is then encoded by multiplying the numbers that represent its letters. For example, CAT is encoded by 3* 1* 20 = 60, MATH is encoded by 13*1*20*8 = 2080. Find a word that would be encoded as 7560 and explain how you found it. Could there be other words? Explain why or why not.

Black Level Problems:

5. What is the least whole number value of x such that $f(x) = x^2 + x + 11$ is not prime?

6. A natural number n, such that $n > 1$, can't be written as the sum of two more consecutive odd numbers if and only if n is prime or n is twice an odd number. Of the ten natural numbers 20 through 29, how many can't be written as the sum of two or more consecutive odd numbers?

Solutions are provided here for the Blue and Black problems:

Blue Solutions

3. The prime factorization of 47,432 is $2^3 x 7^2 x 11^2$. The sum of the three prime factors is 2 + 7 + 11 = 20.

4. LINE, ALIEN, REGAL, BORN, LARGE, BARON are only some of the many.
We began to solve the equation by making a factor tree. We factored the number 7560 until we received four numbers that translated into letters, which we formed into a real word. This is an example of our factor tree:

```
                              7560
                               / \
                             20  378
                             /\  / \
Numbers used:                4 5 18 21
Translation of numbers:      D E  R U
Letters unscrambled:         R U  D E
```

Black Solutions

5. Value of the function can be checked with a chart.

x	$x^2 + x + 11$
0	11
1	13
2	17
3	23
4	31
.	.
.	.
.	.
10	121

When $x = 10$, the value of the function is $x^2 + x + 11 = 10^2 + 10 + 11 = 121$. This is the first value of the function which is not prime, so the answer is 11.

6. The primes from 20 to 29 are 23 and 29. The numbers that are twice an odd number are 22 and 26. These 4 natural numbers can't be written as the sum of two or more consecutive odd numbers.

7^{th} Grade Unit: Operations with Fractions

Lesson: "Combo Day"

Goal: Students are able to identify which operation is needed to solve a problem. They can multiply, divide, add and subtract fractions.

Green level problems:

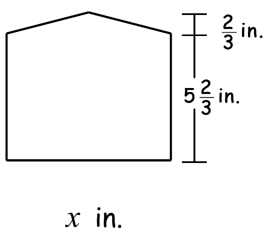
1. How many $\frac{3}{4}$ inch-long beads are needed to make a 12-inch long necklace?

2. You and a friend make a bet. He bets that he is more than $\frac{5}{16}$ inches taller than you. You bet that he is not. If you are 5 ft. $5\frac{5}{8}$ inches tall and he is 5 ft. $5\frac{7}{16}$ inches tall, who won the bet?

Blue Level Problems:

3. Ann gave $\frac{1}{4}$ of her beads to Joey, $\frac{3}{16}$ of them to May and $\frac{1}{5}$ of the rest to Rose. If Ann had 36 beads left, how many beads did she have at first?

4.

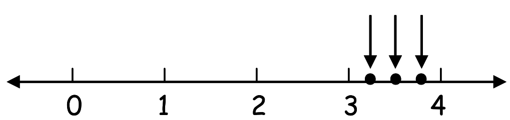


The figure is composed of a triangle and a rectangle. The area of the figure is 24 square inches. Find the value of x.

Black Level Problems:

5. **The Density Property.** The diagram shows a number line. Between 3 and 4 there are many other numbers such as 3.5, 3.79, 3.22861154. . . , etc. If a set of numbers has the property that between any two of them there is another number of that kind, then the set of numbers is said to be *dense.* The work you have been doing with fractions allows you to tell whether or not certain sets of numbers are dense.

There are real numbers between 3 and 4.



Answer the following questions.

a. Find a real number between 4.8 and 4.9.

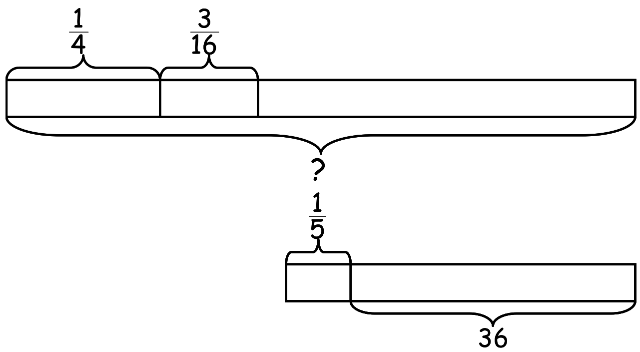
b. Find five real numbers between 7.83 and 7.84.

6. $\frac{2x-11}{x^2-7x+12}+\frac{3}{(x-4)}$

Solutions are provided here for the Blue and Black problems:

<u>Blue Solutions:</u>

3.



After giving part of her beads to Joey and May, fraction of Ann's beads left $= 1 - \frac{1}{4} - \frac{3}{16} = \frac{16}{16} - \frac{4}{16} - \frac{3}{16} = \frac{9}{16}$

After giving part of her beads to Rose, fraction of Ann's beads left $\left(1 - \frac{1}{5}\right) \bullet \frac{9}{16} = \frac{4}{5} \bullet \frac{9}{16} = \frac{9}{20}$

Number of beads Ann had at first= $36 \div \frac{9}{20}$

$$= 36 \bullet \frac{20}{9} = 80$$

She had **80** beads at first.

4. x = 4

<u>Black Solutions</u>:

5. a) Answers may vary. 4.85 **b.** Answers may vary. 7.831, 7.833, 7.835

6. $\frac{5}{x-3}$

7th Grade Unit: Ratios, Proportions, Percents and Probability

Lesson: Proportions

Goals: Students extend their work with ratios to develop an understanding of proportionality and they use proportions to solve problems.
Students use proportions to make estimates relating to a population on the basis of a sample.

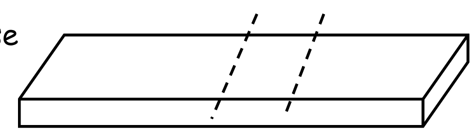
Green level problems:

1. The distance traveled by a bus at a constant speed is directly proportional to the length of time it travels. If a bus travels 320 km in 4 h, how far will it travel in 9h?

2. Of 75 pairs of jeans, 7 have flaws. Estimate how many of 24,000 pairs of jeans are flawed.

Blue Level Problems:

3. A 12-foot board is cut into three pieces whose lengths are in the ratio 3:1:2. How many inches are in the length of the shortest piece?



4. Solve the following proportion for x: $\frac{3x-2}{4}=\frac{x+4}{2}$ ______

Black Level Problems:

5. A wheel rotates once a second. Object two (O) is twice the distance from the center of the wheel as object one (O_1). How fast is O moving in comparison to O_1?

6. Three miners can load 350 pounds of coal in 15 minutes. How many minutes would it take two miners to load 700 pounds of coal?

Solutions are provided here for the Blue and Black problems:

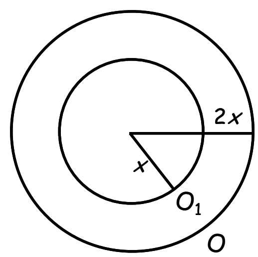
Blue Solutions:

3. The three pieces are in ratio 3:1:2. Since 3 + 1 + 2 = 6, then a 6-foot board would be cut into pieces that were 3, 1, and 2 feet, respectively. Since the board you are given is 12 feet, or twice as long, the piece should each be twice as long, and the smallest piece will be 2 feet.
Since the question asks for the number of *inches* in the smallest piece, the 2 feet must be converted to inches: 2 feet x 12 inches/foot = **24 inches**.

4. 10

Black Solutions:

5. Let x be the distance from the center of the wheel O_1. In one second O_1 goes a distance $2\pi x$. The distance O goes in the same second is $2\pi(2x)$ or twice the distance O_1 goes. Hence, O goes twice the speed of O_1.



6. If three miners can load 350 pounds in 15 minutes, then three miners can load 2 • 350 = 700 pounds in 2 • 15 = 30 minutes. Then, 1 miner can load 700 pounds in 3 • 30 = 90 minutes, and 2 • 1 = 2 miners can load 700 pounds in 90 ÷ 2 = **45 minutes**. Remember, double the number of miners can load double the number of pounds in the same time, or double the number of miners can load the same number of pounds in half the time.

7th Grade Unit: Solving Multi-Step Equations
Lesson: Solving Multi-Step Equations

Goals: Students combine like terms and use the distributive property to simplify and then solve multi-step equations.
Students apply these skills to consecutive integer word problems

Green level problems:

1. Solve. Check your solution. $16 = 2(x-1) - x$

2. The sum of three consecutive integers is 96. Find the integers.

Blue Level Problems:

3. The sum on an odd integer and twice its consecutive is equal to equal 3757. Find the number.

4. **Membership Boom.** There was a surge in math club membership after the fun Halloween party. When the principal asked Mrs. Germain how many students were currently in the club, she replied, "Three times our number plus a third of our number plus a fourth of our number plus you and me is two hundred sixty."

   The principal smiled and nodded, then added, "If you can pick up another eight students before your Christmas party you will have exactly 10% of the student body involved in the club."

   a. How many students are enrolled in the math club?
   b. How many students are in the school?

Black Level Problems:

5. Solve for a in terms of b if two more than twice a is three less than the square of the number which is one less than b.

6. The units of a two-digit number is 40% of the tens digit. If the digits are reversed, the resulting number is 27 less than the original number. Find the original number.

Solutions are provided here for the Blue and Black problems:

Blue Solutions:

3. The difference between two odd integers is equal to 2. let x be an odd integer and x + 2 be its consecutive. The sum of x and twice its consecutive is equal to 3757 gives an equation of the form

$x + 2(x + 2) = 3757$

Solve for x $\quad x = 1251$

Check that the sum of 1251 and 2(1251 + 2) is equal to 3757.

4. Let n = the number of students in the math club.

$3n + \frac{1}{3}n + \frac{1}{4}n + 2 = 260$

$3\,\frac{7}{12}n + 2 = 260$

$3\,\frac{7}{12}n = 258$

$n = 72$

Then, if the 72 math club students gain 8 more members before the Christmas pageant they would have a total of 80 members in the Math Club, and 10% of the schools students, also. 72 + 8 = 80 members.

Black Solutions:

5.

Translating the English into algebra, we get the following equation: $2a + 2 = (b-1)^2 - 3$. Solving for a in terms of b, we get $2a + 2 = (b^2 - 2b + 1) - 3$, then $2a = b^2 - 2b - 4$, and finally $a = \frac{b^2 - 2b - 4}{2}$

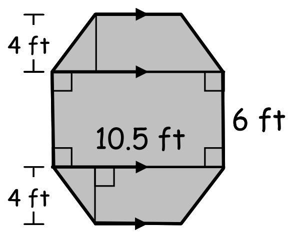
6. 52

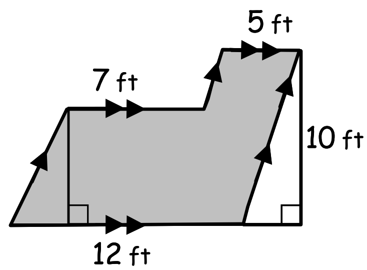
7th Grade Unit: Area and Volume:
Lesson: Area of Polygons

Goal: Students understand and apply area formulas for polygons. Students decompose irregular two dimensional shapes into smaller component shapes.

Green level problems:

1. An architect has the option of the two rooms shown for a new hotel. Which room would provide the most space for the guests?





2. Ken and Kathy went horseback riding at noon. Ken rode north at 3 km per hour and Kathy rode west at 3.5 km per hour. How far apart were they at 2 P.M.? Round to the nearest kilometer.

Blue Level Problems:

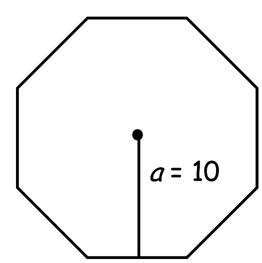
3. **Change in Area.** The length of a rectangle is increased by 20 percent, and its width is decreased by 10 percent. By what percent does the area of the rectangle change? What if, instead, we decreased the length by 20 percent and increased the width by 10 percent?

4. A square blanket measuring x feet by x feet was folded in half, folded in half again and finally folded in half one last time. After these three successive folds, without ever unfolding, the blanket covers an area of 8 square feet. What is the value of x?

Black Level Problems:

5. The ratio of the length of the base to the length of the height of the triangle is 2 : 3. The area of the triangle is 81 square centimeters. What is the number of centimeters in the height of the triangle? Express your answer in simplest radical form.

6. Find the area of a regular octagon with side 20 and apothem 10



Solutions are provided here for the Blue and Black problems:

Blue Solutions:

3. **Change in Area.**

Eight percent increase in the first case and 12 percent decrease in the second case. Let L represent the length and W represent the width, so LW represents the area. If L is increased by 20 percent, it becomes $1.2L$. If W is decreased by 10 percent, it becomes $0.9W$. The area of the new rectangle is $(1.2L)(0.9W) = 1.08\ LW$, which is an 8 percent increase over the original LW. In the second case, we have a new area of $(0.8L)(1.1W) = 0.88\ LW$, which accounts for a 12 percent decrease from LW.

4. If 8 square feet is half of half of half of the area of the blanket, then the area of the blanket is 8 x 2 x 2 x 2 = 64 square feet. The side length x, must be square root of 64, which is **8** feet.

Black Solutions:

5. Given the ratio 2 : 3, we can write the base as two times as unknown value x and the height as three times the same unknown value x. The area formula for a triangle is $A = \frac{1}{2}bh$. We know the area and can write the height and base in terms of x. This gives us: $81 = \frac{1}{2}(2x)(3x)$, which simplifies to $81=3x^2$. Dividing both sides by 3, we get $27 = x^2$. Now we can take the square root of both sides and we get: $x = \sqrt{27} = \sqrt{9 \times 3} = 3\sqrt{3}$. The height is three times this amount, or $3 \times 3\sqrt{3} = 9\sqrt{3}$ centimeters.

6. In this case the perimeter of the octagon is 160. The area of the octagon is $A = \frac{1}{2}ap = \frac{1}{2} \bullet 160 \bullet 10 = 800$.

Grade 8 Goal: Evaluate Expressions

Green

Substitute and evaluate the algebraic expressions if m = -2, n = 3, s = 5, p = $\frac{4}{5}$, and y = $\frac{1}{3}$

$$-9y^2 + 15p - 15$$

Blue

Let $y_1 = \frac{x-1}{x+1}$. Let y_2 by the simplified expression obtained by replacing x in y_1 by $\frac{x}{3}$. Let y_3 be the simplified expression obtained by replacing x in y_2 by $\frac{x}{3}$, and so forth. Evaluate y_4 when $x = 0$

Black

Given the positive integers w, x, y, z with $\frac{w}{x} < \frac{y}{z} < 1$; arrange in order of increasing absolute value the five quantities: $\frac{x}{w}$, $\frac{z}{y}$, $\frac{xz}{wy}$, $\frac{x+z}{w+y}$, 1

Blue Solution

7. Let $y_1 = \dfrac{x-1}{x+1}$. Let y_2 by the simplified expression obtained by replacing x in y_1 by $\dfrac{x}{3}$. Let y_3 be the simplified expression obtained by replacing x in y_2 by $\dfrac{x}{3}$, and so forth. Evaluate y_4 when $x=0$ (2 points)

$$y_2 = \frac{\frac{x}{3}-1}{\frac{x}{3}+1} = \frac{\frac{x-3}{3}}{\frac{x+3}{3}} = \frac{x-3}{3}\cdot\frac{3}{x+3} = \frac{x-3}{x+3}$$

$$y_3 = \frac{\frac{x}{3}-3}{\frac{x}{3}+3} = \frac{\frac{x-9}{3}}{\frac{x+9}{3}} = \frac{x-9}{x+9}$$

$$y_4 = \frac{\frac{x}{3}-9}{\frac{x}{3}+9} = \frac{x-27}{x+27}$$

when $x=0$ $\quad y_4 = \dfrac{0-27}{0+27}$

$y_4 = \dfrac{-27}{27}$

$\boxed{y_4 = -1}$

Black Solution

6. Given the positive integers w, x, y, z with $\frac{w}{x} < \frac{y}{z} < 1$; arrange in order of increasing absolute value the five quantities: $\frac{x}{w}, \frac{z}{y}, \frac{xz}{wy}, \frac{x+z}{w+y}, 1$ (3 points)

Since $\frac{y}{z} < 1$, $\frac{z}{y} > 1$. Since $\frac{w}{x} < \frac{y}{z}$, $\frac{x}{w} > \frac{z}{y}$.

Hence, $1 < \frac{z}{y} < \frac{x}{w}$. Since each of $\frac{z}{y}$ and $\frac{x}{w}$ is greater than 1, Then $\frac{xz}{wy}$ is greater than either.

Hence, $1 < \frac{z}{y} < \frac{x}{w} < \frac{xz}{wy}$

AND Since $\frac{z}{y} < \frac{x}{w}$ $zw < xy$; Therefore, $yz + wz < yz + xy$ and $z(y+w) < y(z+x)$. $\frac{z}{y} < \frac{x+z}{w+y}$. Also, since $xy > wz$, $wx + xy > wx + wz$, $x(w+y) > w(x+z)$, and Therefore $\frac{x+z}{w+y} < \frac{x}{w}$

So, the order is $1, \frac{z}{y}, \frac{x+z}{w+y}, \frac{x}{w}, \frac{xz}{wy}$

Grade 8 Goal: Problem Solve with Linear Equations

Green

Jennifer is 5 years younger than Amanda. If the sum of their ages is 51, how old is each of them?

Blue

Tiffany's Triumphs: Tiffany plays first board for her middle school chess team. Since she joined the team last year, she has won 27 of 51 tournament games. That's a winning percentage of about 53%. Winning a lot of matches in a row is pretty unlikely. Let's say that Tiffany gets hot and wins two out of every three games she plays. How many more games will Tiffany have to play before she has a winning percentage of 60%?

Black

A merchant on his way to the market with n bags of flour passes through three toll gates. At the first gate, the toll is $\frac{1}{8}$ of his holdings, but 4 bags are returned. At the second gate, the toll is $\frac{1}{4}$ of his (new) holdings, but 3 bags are returned. At the third gate, the toll is $\frac{1}{3}$ of his (new) holdings, but 1 bag is returned. The merchant arrives at the market with exactly $\frac{n}{2}$ bags. If all transactions involve whole bags, find the value of n.

Blue Solution

3. **Tiffany's Triumphs**: Tiffany plays first board for her middle school chess team. Since she joined the team last year, she has won 27 of 51 tournament games. That's a winning percentage of about 53%. Winning a lot of matches in a row is pretty unlikely. Let's say that Tiffany gets hot and wins two out of every three games she plays. How many more games will Tiffany have to play before she has a winning percentage of 60%?

① Let x = the additional games Tiffany plays, $\frac{2}{3}x$ = the games she wins out of those additional games.

② $$\frac{27+\frac{2}{3}x}{51+x} = \frac{60}{100}$$

③ Cross multiply since cross products in any proportion are equal.

$$2700 + \frac{200}{3}x = 3060 + 60x$$

$$66\tfrac{2}{3}x - 60x = 360$$

$$6\tfrac{2}{3}x = 360$$

$$\frac{20}{3}x = 360$$

$$x = 54$$ Tiffany must play 54 more games

④ $\frac{2}{3}$ of 54 is 36. So, if Tiffany plays 54 more games, she'll have a total of 27 + 36 wins = 63 wins and 51 + 54 = 105 total games. That percent is $\frac{63}{105} \cdot 100 =$ 60% ✓

Black Solution

3. A merchant on his way to the market with n bags of flour passes through three toll gates. At the first gate, the toll is $\frac{1}{8}$ of his holdings, but 4 bags are returned. At the second gate, the toll is $\frac{1}{4}$ of his (new) holdings, but 3 bags are returned. At the third gate, the toll is $\frac{1}{3}$ of his (new) holdings, but 1 bag is returned. The merchant arrives at the market with exactly $\frac{n}{2}$ bags. If all transactions involve whole bags, find the value of n.

The # of bags remaining after the first toll is $n - \frac{1}{8}n + 4 = \frac{7}{8}n + 4$

After the 2nd toll, $\frac{3}{4}\left(\frac{7}{8}n + 4\right) + 3 = \frac{21}{32}n + 3 + 3 = \frac{21}{32}n + 6$

After the 3rd toll, $\frac{2}{3}\left(\frac{21}{32}n + 6\right) + 1 = \frac{7}{16}n + 5$

$$\frac{7}{16}n + 5 = \frac{n}{2}$$

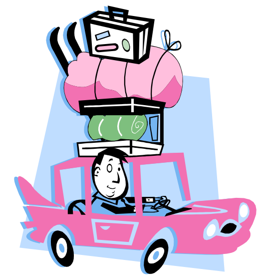
$$5 = \frac{1}{16}n$$

$$80 = n$$ 80 bags of flour

Grade 8 Goal: Solve Distance, Rate, Time problems

Green

Two cars travel the same distance. The first car travels at a rate of 50 miles per hour and reaches its destination in t hours. The second car travels at a rate of 60 miles per hour and reaches its destination 2 hours earlier than the first car. How long does it take the first to reach its destination?



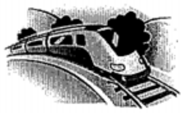
Blue

Two trains started from the same point. At 11 p.m., the first train traveled East at the rate of 50 mph. At 9:00 a.m. the next morning, the second train traveled West at the rate of 140 mph. At what time were they 1260 miles apart?

Black

At what time after 7:00 will the minute hand overtake the hour hand?

Blue Solution



4. Two trains started from the same point. At 11 p.m., the first train traveled East at the rate of 50 mph. At 9:00 a.m. the next morning, the second train traveled West at the rate of 140 mph. At what time were they 1260 miles apart?

2nd Train West @ 140 mph ← START → 1st Train East @ 50 mph

① Let t = time travelled by the 1st train
$t-10$ = time travelled by the 2nd train

② $D_{TOTAL} = D_{1st\ Train} + D_{2nd\ train}$

$$1260 = \overset{50}{50t} + 140(t-10)$$

③ $1260 = 50t + 140t - 1400$

$2660 = 190t$

$14 \text{ hours} = t$

9 a.m. + 14 hours = 1 p.m.

$\boxed{\text{They were 1260 miles apart at 1 p.m.}}$

4. Check

1st train travelled
14 hours × 50 mph = 700 miles

2nd train travelled
4 hours × 140 mph = 560 miles

700 + 560 =
1260 miles
apart altogether
✓

Black Solution

1. At what time after 7:00 will the minute hand overtake the hour hand?

Let r = the speed of the hour hand
$12r$ = the speed of the minute hand
x = the distance the minute hand must travel (in minutes)
$x-35$ = the distance the hour hand must travel (in minutes)

When the 2 hands meet, $\text{Time}_{\text{HOUR HAND}} = \text{T}_{\text{MINUTE HAND}}$

$$\frac{x-35}{r} = \frac{x}{12r}$$

$$\frac{12r(x-35)}{r} = \frac{xr}{r}$$

$$12(x-35) = x$$

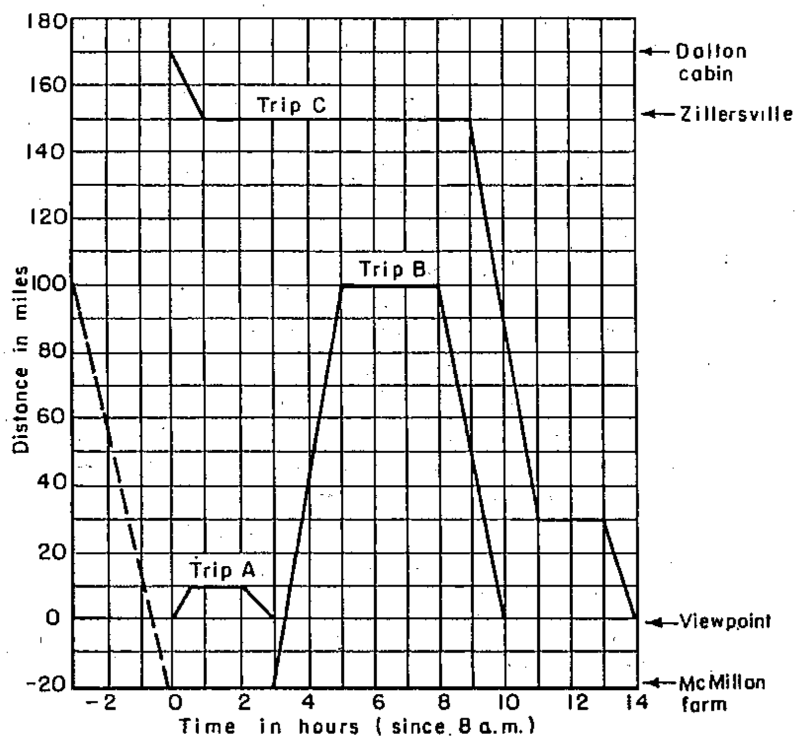
$$\frac{11x}{11} = \frac{420}{11}$$

$x = 38\frac{2}{11}$ so, the minute hand will overtake the hour hand at $7{:}38\frac{2}{11}$

Grade 8 Goal: Analyze Change in Various Contexts (Interpret Story Graphs)

Green

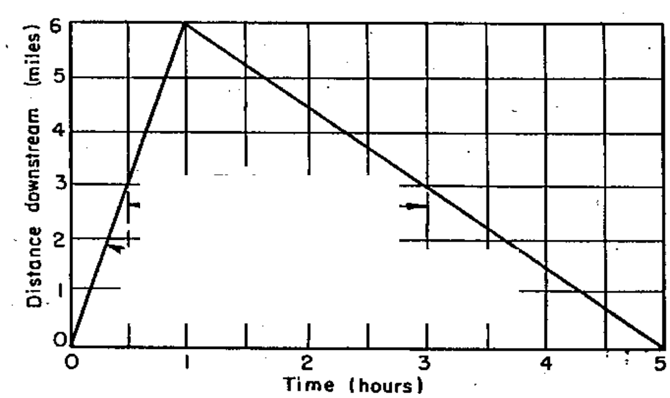
1. The lines on the following graph represent three trips taken by three different people on the same day. Use the graph to fill in the blank spaces in the description of trip A.



Trip A – With his pencils and sketch pad in the bicycle basket, Mr. Lewis left Viewpoint at 8 a.m. and rode his bicycle for ___ hour(s) into the country at a speed of ____ miles per hour. He stayed in the country for ___ hour(s) sketching and drawing. He rode back to Viewpoint on his bicycle in ____ hour(s) at a speed of ____ miles per hour.

2. Steve can row a boat 6 miles per hour going downstream (in the same direction as the force of the current) and 1 ½ miles per hour going upstream (in the opposite direction of the current). Steve has only 5 hours to complete a round trip downstream.

   Use the following graph to answer the questions that follow:



A. How many hours after he begins his trip must he start his return upstream?

B. How far downstream can he go? Explain.

C. If he stops at a dock 3 miles downstream, how long may he fish before he starts back? **Explain** your answer.

Grade 8 Goal: Analyze Change in Various Contexts (Interpret Story Graphs)

Blue

1. A motor boat traveling at the rate of 30 miles per hour crossed the width of a lake in 25 minutes less than when traveling at the rate of 20 miles per hour. What is the width of the lake? Create a graph that represents this story and show how the answer to this question can be seen on this graph.

2. A soldier fired a bullet which has a speed of 1500 feet per second. One and a quarter seconds later he heard it strike the target. Sound travels at approximately 1000 feet per second. How far away was the target from the soldier? Create a graph that represents this story and show how the answer to this question can be seen on this graph.

Black

1. Two trains start out on the same track toward each other. Train A travels at 30 miles per hour. Train B travels at 40 miles per hour. At the start, they are 700 miles apart.

   When train A starts, a fly that was resting on the headlight starts to fly down the track ahead of the train at a speed of 100 miles per hour. When it meets train B, it turns and flies back along the track toward train A.

   The fly continues to fly back and forth along the track at 100 miles per hour, reversing its direction each time it meets one of the trains, until it is crushed between the colliding trains.

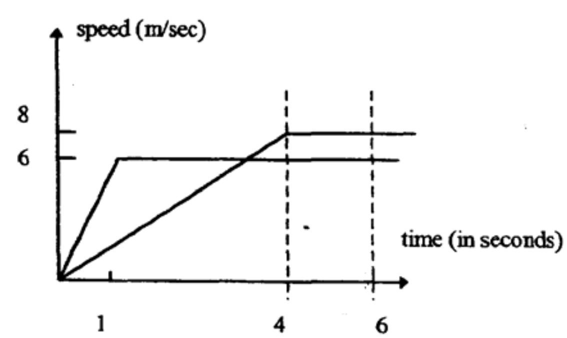
   A. On the same graph, show the motion of the two trains.

   B. Also show the motion of the fly

   C. How many miles did the fly travel before it was crushed? Explain how you found your answer.

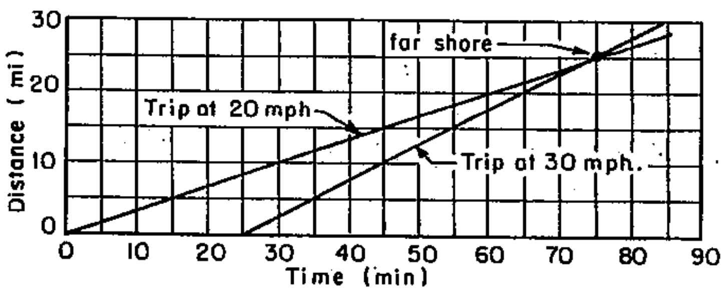
2. The following figure describes the first six seconds of two cars that are racing down a straight race track. Each car continues the rest of the race at the same speed as it is moving after the first four seconds. One car moves ahead first, and then the second car catches up.

   How many seconds will pass before the second car catches up to the first?

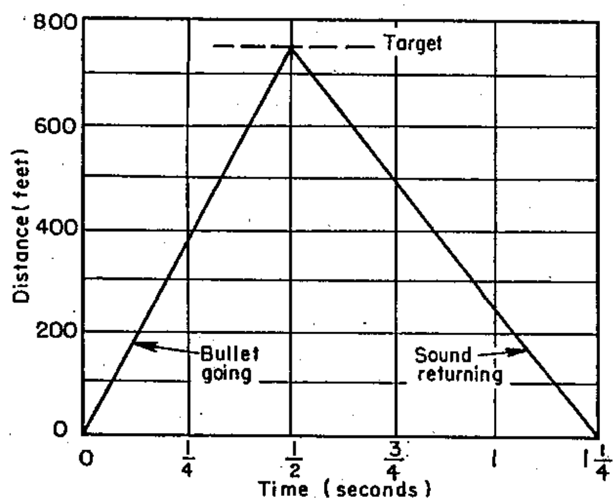


Blue Solutions

(When the boat travels 20 miles per hour, it can reach the opposite shore of the lake at a certain time. When it travels 30 miles per hour, it can leave 25 minutes later and still reach the opposite shore at the same time. The line for the slower trip starts at the origin. Its slope can be laid out by going 60 (minutes) to the right and 20 (miles) upward. The line for the faster trip starts at the point 25,0. Its slope can be laid out by going 60 to the right and 30 upward. From the graph the width of the lake is 25 miles.)

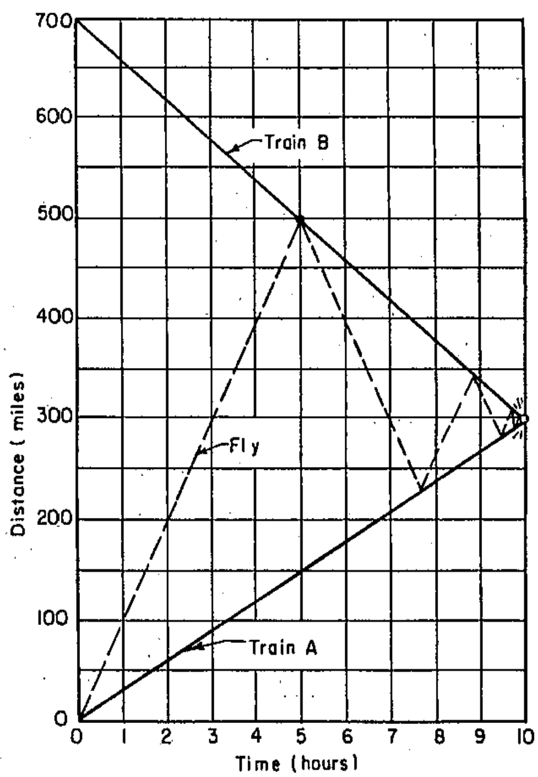


(The line for the bullet's progress starts at 0,0 and has a slope of 1500. The line for the sound of the bullet striking the target ends at $1\frac{1}{4}$,0 and has a slope of –1000. The graph shows that the target was 750 feet away.)



Black Solutions

(The line for train A starts at 0,0 and has a slope of 30. The line for train B starts at 0,700 and has a slope of –40. The slope of the line for the fly alternates between 100 and –100, changing its sign each time it reaches the line for one of the trains. The graph shows that the fly flew for 10 hours before it was crushed. Since its speed was 100 miles per hour, it flew $10 \times 100 = 1000$ miles altogether.)



Let t = the time that each car has travelled since the beginning of the race.

When the cars meet, the distances they will have travelled will be equal.

Distance (Car 1) = Distance (Car 2)

$$3 + 6(t-1) = 16 + 8(t-4)$$

$$3 + 6t - 6 = 16 + 8t - 32$$

$$6t - 3 = 8t - 16$$

$$13 = 2t$$

$$\boxed{6.5 = t}$$

seconds

Grade 8 Goal: Graph a linear function by using a table of values

Green

Rewrite the following equation in function form. Then, use
a table of values to graph the function.

$9x - 3y = 15$

Blue

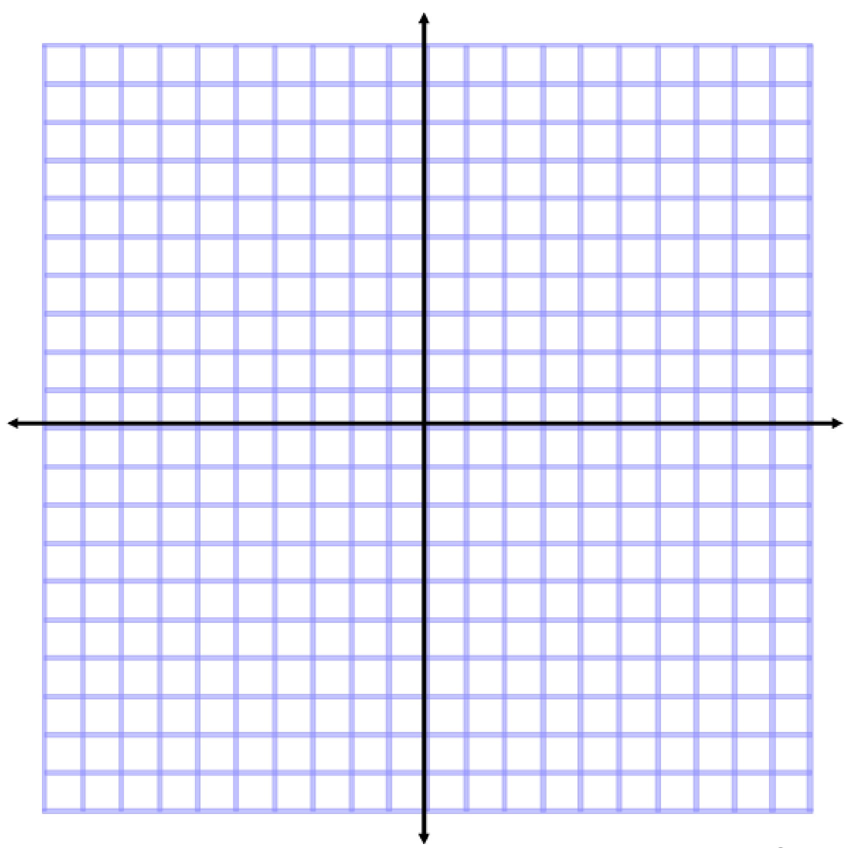
Use a table of values to graph the functions mentioned
in the following problem. Then, answer the question.

What is the number of square units in the area of the region
bounded by the graphs of $y = 2|x|$ and $y = -2|x| + 4$?

Black

Consider the step function [x] = y.
Define it by y = the value of the greatest integer less than or equal to x.
Use a table of values to graph the function

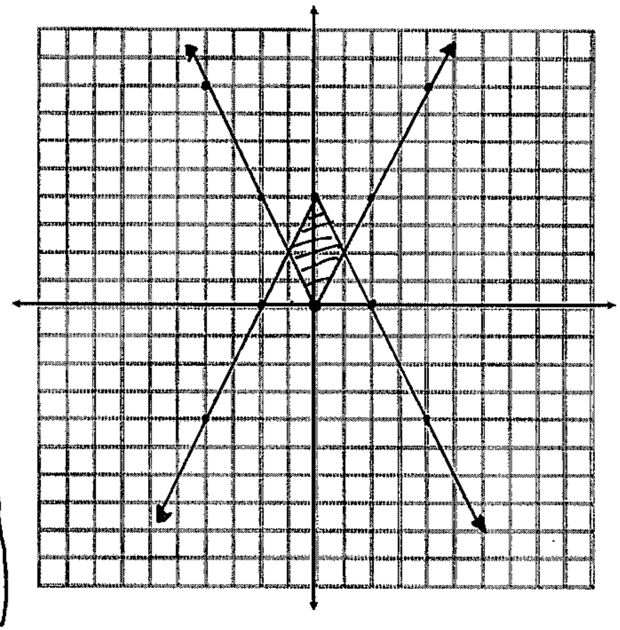
y = [x-1]



Blue Solution

7. Use a table of values to graph the equations mentioned in the following problem. Then, answer the question.

   What is the number of square units in the area of the region bounded by the graphs of $y = 2|x|$ and $y = -2|x| + 4$ (3)



$y = 2|x|$

x	y
-4	8
-2	4
0	0
2	4
4	8

$y = -2|x| + 4$

x	y
-4	-4
-2	0
0	4
2	0
4	-4

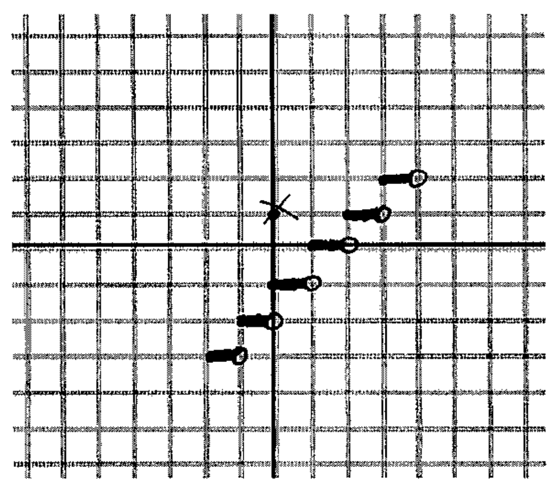
AREA = 4 square units

Black Solution

Consider the step function [x] = y. Define it by y = the value of the greatest integer less than or equal to x. Use a table of values to graph the function (3 pts)

y = [x-1]

x	y
0	-1
.5	-1
1	0
1.5	0
2	1
2.5	1
3	2
-.5	-2
-1	-2
-1.5	-3



Grade 8 Goal: Graph horizontal and vertical lines

Green

Use a table of values to graph the functions.

$y = -5$, $x = 6$

Blue

The point $(5, 3)$ is reflected about the line $x = 2$. The image point is then reflected about the line $y = 2$. The resulting point is (a, b). Compute $a + b$.

Blue and Black

DESCARTES' TRIANGLE

Two vertices of a triangle are located at (0,6) and (0,12). If the area of the triangle is 12 square units

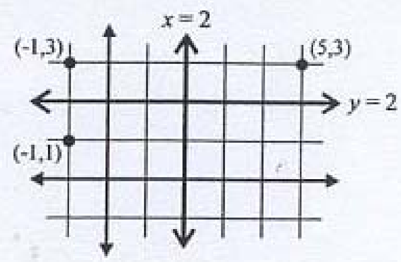
BLUE: What are all the possible positions for the third vertex?

BLACK: What if the triangle is also isosceles?

Solutions

Blue

Reflection of $(5,3)$ over $x=2$ results in image point $(-1,3)$. Reflection of $(-1,3)$ over $y=2$ results in image point $(-1,1)$. The sum of the coordinates of this point is $-1+1=0$.



BLUE
Since the formula for area of a triangle is bh/2, 6h/2=12, h must be 4. therefore, on the coordinate plane, since one segment of the triangle is on the y axis, the x coordinate must merely move 4 spaces to the left or right (making it -4 or 4) therefore, any point on the line x=4 or on the line x=-4 will satisfy the equation.

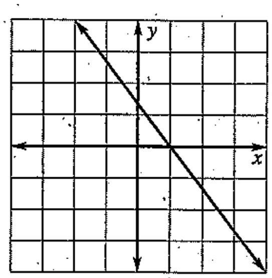
BLACK
In order to make the triangle isosceles, one of two things must happen: the two sides other than the one with the side length 6 can be equal OR one other side can equal 6 as well. The key in this is that THE AREA MUST BE 12. So, the two points that will satisfy the first part are (4,9) , (-4,9). This is because the x coordinate must be plus or minus 4, and the y coordinate will be the average of those on the endpoints of the given side. In the other scenario, another side must equal 6. the side could be above the line or below the line, and on either side of the y axis. that makes four points. since the side length must be 6, and the distance from the y axis is 4, a right triangle can be made. using Pythagorean, 6^2-4^2=20, the root of which is 2(root5). Therefore, the y coordinates will be the bottom coordinate MINUS 2root5 OR the top coordinate PLUS 2root5. So, the y coordinates will be the lower (6) minus 2root5 and the higher (12) plus 2root5. since the x coordinates will be plus or minus 4, the last 4 coordinates will be: (4,6-(2root5)) , (4,12+(2root5)) , (-4,12+(2root5)) , (-4,6-(2root5)). Therefore, the 6 points are: (4,9) , (-4,9) , (4,6-(2root5)), (4,12+(2root5)) , (-4,12+(2root5)) , (-4,6-(2root5))

Grade 8 Goal: Calculate the slope between two points on the coordinate plane

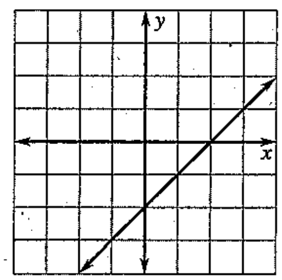
Green

1. What is the slope of each line pictured below?

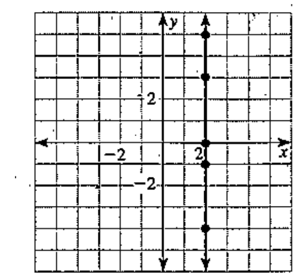
a.



b.



c.



2. Calculate the slope of the lines passing through the following pairs of points:

(-10,-7) and (1,-2)

Blue

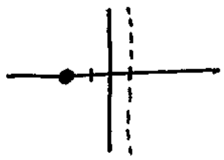
If (-2,0) and (1,s) are two points on a line with a negative slope, what can you say about s?

Black

If $a < 5$, what must be true about b so that the line passing through the points (a, b) and (1, -3) has a negative slope?

Blue Solution

If (-2,0) and (1,s) are two points on a line with a negative slope, what can you say about s? (2 pts)

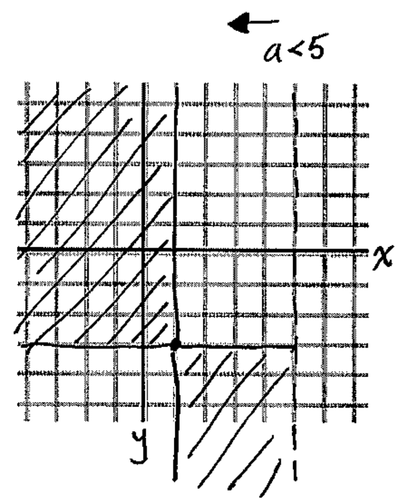


For the slope to be negative, $(1, s)$ must be on the dotted line below the x-axis.

Therefore, $\boxed{s < 0}$

Black Solution

If $a < 5$, what must be true about b so that the line passing through the points (a, b) and (1, -3) has a negative slope? (2 pts)



(a, b) can be in either shaded area.

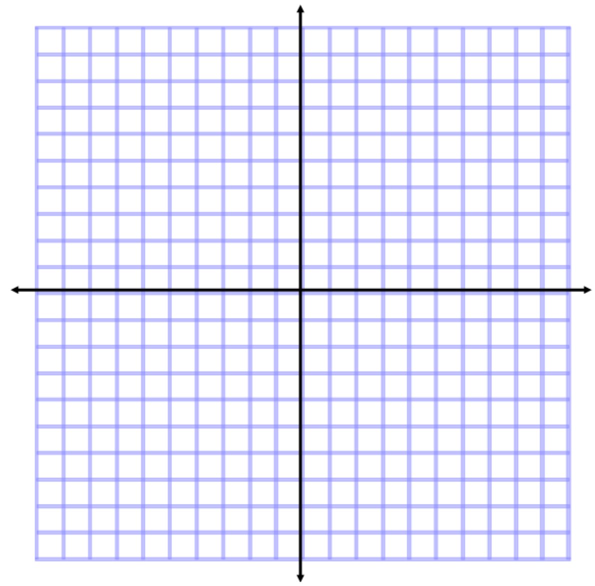
So, if $1 < a < 5$, then $b < -3$.

if $a < 1$, $b > -3$.

Grade 8 Goal: Problem Solve with Graphs of Linear Functions

Green

Your class is selling shirts and sweaters displaying your school logo to raise money for a field trip. Your class needs to raise $1800 to cover the cost of the trip. For each shirt sold, $3 is raised for the trip. For each sweater sold, $5 is raised for the trip.



A. Write an equation that represents the number of shirts (x) and sweaters (y) sold and the amount of money your class needs to raise for the field trip.

B. Find the x-intercept. What does it represent?

C. Find the y-intercept. What does it represent?

D. Graph the equation.

Blue

Steve is making crafts to sell at a fair. It takes him $\frac{2}{3}$ of an hour to make a wooden fork and $\frac{1}{2}$ hour to make a wooden spoon. He has 6 hours to work.

A. Write an equation to show the relationship between how many forks and spoons Steve can make in three hours. Let x = the number of forks and y = the number of spoons.

B. Graph your equation from Part A.

C. What is the x-intercept? What does it represent in this situation?

D. What is the y-intercept? What does it represent in this situation?

E. If Steve works the entire 6 hours while making 8 spoons, how many forks did he make?

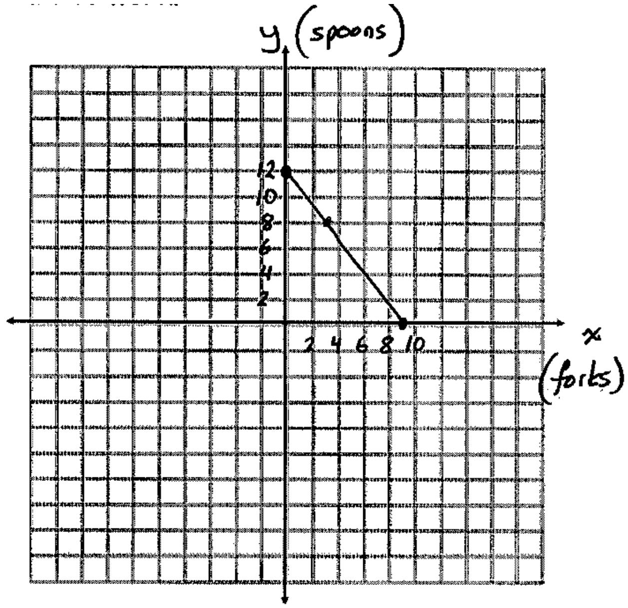
Black

You have $12 to spend on fruit for a meeting. Grapes cost $1 per pound and peaches cost $1.50 per pound. Let x represent the number of pounds of grapes you can buy. Let y represent the number of pounds of peaches you can buy. Write and graph an inequality to model the amounts of grapes and peaches you can buy.

Blue Solution

A. Write an equation to show the relationship between how many forks and spoons Steve can make in three hours. Let x = the number of forks and y = the number of spoons. (2)

$\boxed{\frac{2}{3}x + \frac{1}{2}y = 6}$



B. Graph your equation from Part A. (2)

when $x=0$ $y=12$; when $y=0, x=9$ →

C. What is the x-intercept? What does it represent in this situation? (1)

$(9,0)$: If he makes 0 spoons, he can make 9 forks

D. What is the y-intercept? What does it represent in this situation? (1)

$(0,12)$ If he makes 0 forks, he can make 12 spoons

E. If Steve works the entire 6 hours while making 8 spoons, how many forks did he make? (1)

$y=8$ so $\frac{2}{3}x + \frac{1}{2}\cdot 8 = 6 \rightarrow \frac{2}{3}x + 4 = 6 \rightarrow \frac{2}{3}x = 2 \rightarrow x = 3$ $\boxed{3 \text{ forks}}$

Black Solution

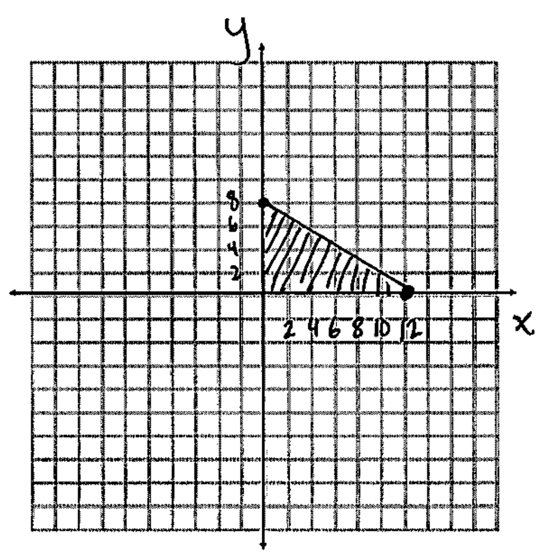
.You have \$12 to spend on fruit for a meeting. Grapes cost \$1 per pound and peaches cost \$1.50 per pound. Let x represent the number of pounds of grapes you can buy. Let y represent the number of pounds of peaches you can buy. Write and graph an inequality to model the amounts of grapes and peaches you can buy. (1 pt)

$\boxed{1x + 1.50y \le 12}$

if $x=0$ $y=8$ $(0,8)$

if $y=0$ $x=12$ $(12,0)$

You can buy a combination that is located anywhere within the shaded region.



Grade 8 Goal: Apply Linear Systems of Equations

Green

You sold two different types of wrapping paper for your fundraiser. One type sold for $6 and the other for $8. You collected $92 for the 14 rolls you sold. How many rolls of each type did you sell?

Blue

Bubba Gump has been pretty hungry lately. Shrimp and Mussels are sold by the pound. Last week, Bubba bought $4 worth of shrimp and $2 worth of mussels for a total weight of 8 pounds. This week, he bought enough for himself and his girlfriend to share. He bought $2 worth of shrimp and $9 worth of mussels for a total weight of 20 pounds. Use a 4 step problem solving process to help Bubba figure out how much he paid per pound for Shrimp and Mussels.

Black

For the problem below, complete the following steps to find the desired solution.

1. Define your unknowns
2. Express the objective function and the constraints
3. Graph the constraints
4. Find the corner points to the region of possible solutions
5. Evaluate the objective function at all the feasible corner points

A gold processor has two sources of gold ore, source A and source B. In order to keep his plant running, at least three tons of ore must be processed each day. Ore from source A costs $20 per ton to process, and ore from source B costs $10 per ton to process. Costs must be kept to less than $80 per day. Moreover, Federal Regulations require that the amount of ore from source B cannot exceed twice the amount of ore from source A. If ore from source A yields 2 oz. of gold per ton, and ore from source B yields 3 oz. of gold per ton, how many tons of ore from both sources must be processed each day to maximize the amount of gold extracted subject to the above constraints?

Blue Solution

Let x = the price per pound for shrimp

y = the price per pound for mussels

Last Week:

(1) $4\left(\frac{1}{x}\right) + 2\left(\frac{1}{y}\right) = 8$

This week:

(2) $2\left(\frac{1}{x}\right) + 9\left(\frac{1}{y}\right) = 20$

Let $u = \frac{1}{x}$ and $v = \frac{1}{y}$

(1) $4u + 2v = 8$

(2) $2u + 9v = 20$

Combine (1) and (2)

$$\begin{array}{r} 4u + 2v = 8 \\ + \; -2(2u + 9v = 20) \\ \hline -16v = -32 \end{array}$$

$v = 2$ so $u = 1$

if $v = 2$, then $y = \$.50$ since $v = \frac{1}{y}$

if $u = 1$, then $x = \$1$ since $u = \frac{1}{x}$

So, shrimp cost \$1 per pound and mussels cost \$.50 per pound

Check:

$4\left(\frac{1}{1}\right) + 2\left(\frac{1}{.5}\right) \stackrel{?}{=} 8$
$4 + 4 = 8$ ✓ } LAST WEEK

$2\left(\frac{1}{1}\right) + 9\left(\frac{1}{.5}\right) \stackrel{?}{=} 20$
$2 + 18 = 20$ ✓ } THIS WEEK

Black Solution

1. let x = the number of tons from source A
   y = the number of tons from source B

2. The objective is to maximize the amount of gold yield (the gold produced). Since each ton of ore from source A yields 2 oz. of gold and each ton of ore from source B yields 3 oz of gold, the amount of gold recovered will be

$$2x + 3y = \text{Gold Yield}$$

Constraints

Processing $x + y \geq 3$

Cost $20x + 10y \leq 80$

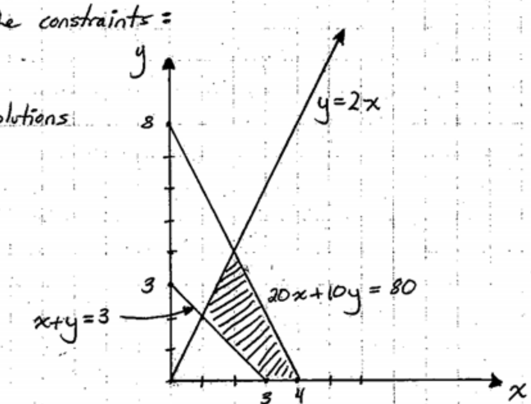
Federal Regulations $y \leq 2x$

A negative number of tons of ore will not be processed

$x \geq 0$ and $y \geq 0$

3. Graph the constraints:

Shade the common solutions



4. Find the feasible corner points: (3,0) (4,0) (1,2) and (2,4)

5. Evaluate the objective function at each corner point:
   $(1,2) \rightarrow 2(1) + 3(2) = 8$ oz of gold
   $(2,4) \rightarrow 2(2) + 3(4) = 16$ oz of gold ★
   $(3,0) \rightarrow 2(3) + 3(0) = 6$ oz of gold
   $(4,0) \rightarrow 2(4) + 3(0) = 8$ oz of gold

   The maximum yield of gold is 16 oz. by processing 2 tons of ore from source A and 4 tons from source B.

Grade 8 Goal: Graph Quadratic Functions

Green

Graph the following equation. Make sure you show your work and label the coordinates of the vertex.

$$y = x^2 - 4x - 5$$

Blue

Your dance class has decided to perform *The Nutcracker*. *The Nutcracker* is one of the most popular Christmas holiday ballets today.

Your class chooses two primary dancers for the lead roles, one male and one female. One of the male dancer's leaps can be modeled by the equation $h = 2t - t^2$ where h is the height in feet and t is the time in seconds. One of the female dancers' leaps can be modeled by the equation $h = 3t - t^2$.

A. Sketch the graphs of the equations for the male and female dancers.

B. What is the maximum height reached by the male dancer when he leaps?

C. How many seconds does it take the male dancer to reach his maximum jump height?

D. What is the maximum height reached by the female dancer when she leaps?

E. How many seconds does it take the female dancer to reach her maximum height?

Black

The arch of a stone bridge has the shape of a parabola that is 20 feet wide at the water line and rises to its highest point 8 feet above the water. Suppose the vertex of the parabola is at the origin of a coordinate system.

A. Write an equation for the parabola. Illustrate this situation on a coordinate plane.

B. If the water level rises to 18 feet below the highest point of the bridge, how wide is the parabola at the water line?

Blue Solution

14. Your dance class has decided to perform *The Nutcracker*. *The Nutcracker* is one of the most popular Christmas holiday ballets today.

Your class chooses two primary dancers for the lead roles, one male and one female. One of the male dancer's leaps can be modeled by the equation $h = 2t - t^2$ where h is the height in feet and t is the time in seconds. One of the female dancers' leaps can be modeled by the equation $h = 3t - t^2$.

A. Sketch the graphs of the equations for the male and female dancers.

MALE

$h = 2t - t^2$

$h = t(2-t)$

Find the time when the male dancer is on the ground by substituting $h=0$

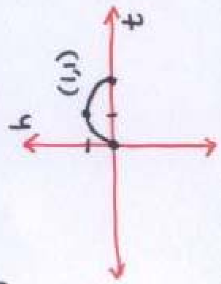
$0 = t(2-t)$

$t = 0$ or 2 — The male is on the ground at zero seconds and after 2 seconds.

Midpoint at $t=1$

$h = 2\cdot1 - 1^2$

$h = 1$



FEMALE

$h = 3t - t^2$

$h = t(3-t)$

The female dancer is on the ground when $h=0$.

$0 = t(3-t)$

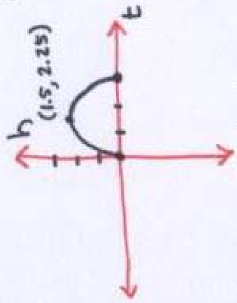
$t = 0$ or 3

midpoint at $t = 1.5$

$h = 3(1.5) - 1.5^2$

$h = 4.5 - 2.25$

$h = 2.25$



B. What is the maximum height reached by the male dancer when he leaps? 1

C. How many seconds does it take the male dancer to reach his maximum jump height? 1 second

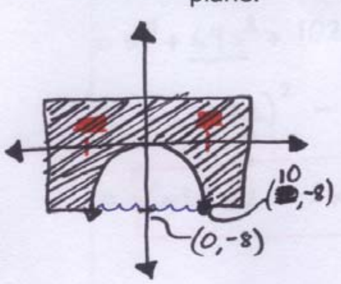
D. What is the maximum height reached by the female dancer when she leaps? 2.25

E. How many seconds does it take the female dancer to reach her maximum height? 1.5 seconds

Black Solution

9. The arch of a stone bridge has the shape of a parabola that is 20 feet wide at the water line and rises to its highest point 8 feet above the water. Suppose the vertex of the parabola is at the origin of a coordinate system.

A. Write an equation for the parabola. Illustrate this situation on a coordinate plane.



The equation of a parabola has the form $y = a(x-p)(x-q)$, where a is a constant and p and q are the x-intercepts.

For this situation:

$y = a(x-0)(x-0)$

$y = ax^2$ ~ $-8 = a\cdot10^2$

[redacted]

$\frac{-8}{100} = a$

$\frac{-2}{25} = a$

$\boxed{y = \frac{-2}{25}x^2}$

B. If the water level rises to 18 feet below the highest point of the bridge, how wide is the parabola at the water line?

$(?, -18)$ $\quad -18 = \frac{-2}{25}x^2$

$\frac{-18\cdot25}{-2} = x^2$

$225 = x^2$ ~ so $x = 15$ or -15.

The water line would go from -15 to 15, a width of $\boxed{30.}$

Grade 8 Goal: Apply Exponential (Growth) Functions

Green

You deposit $500 into an account that pays 8% interest compounded yearly. Write a function to represent your account's balance B after x years. What will the account balance be after 6 years?

Blue

Copying Conundrum: We have a copy machine in the office that will let you increase or decrease the size of a picture by a certain percentage. For example, if I have a picture that is 4 inches by 6 inches and I copy it at 50%, the dimensions on the copy will be 2 inches by 3 inches.

Start with an 8-inch by 10-inch picture. Copy it at 80%, then copy the copy at 80%, and continue this process until you have 5 copies of the original, each of which is smaller than the previous one. Write a formula that will allow you to easily find the picture's largest dimension after doing this n times.

Black

There are 100 rats in a given population. At first, and under ideal circumstances, the rat population increases 20% per month. After 2 months, some bait is laid that contains a substance that slows their population growth. For one month the population remains constant. After that, the population starts to decrease by 50% per month.

There are three parts to this story. Find the function that matches each part of the story (if your function contains decimal values, round those values to the nearest whole number). Also, create a single graph that illustrates all three parts of this story.

Blue Solution

7. Copying Conundrum: We have a copy machine in the office that will let you increase or decrease the size of a picture by a certain percentage. For example, if I have a picture that is 4 inches by 6 inches and I copy it at 50%, the dimensions on the copy will be 2 inches by 3 inches.

Start with an 8-inch by 10-inch picture. Copy it at 80%, then copy the copy at 80%, and continue this process until you have 5 copies of the original, each of which is smaller than the previous one. Write a formula that will allow you to easily find the picture's largest dimension after doing this n times.

copy #	largest dimension
0	10
1	10 × .8
2	10 × .8 × .8

$\boxed{\text{largest dimension} = 10(.8)^n}$

Black Solution

6. There are 100 rats in a given population. Under ideal circumstances, at first the rat population increases 20% per month. After 2 months, some bait is laid that contains a substance that slows their population growth. For one month the population remains constant. After that, the population starts to decrease by 50% per month.

There are three parts to this story. Find the function that matches each part of the story (if your function contains decimal values, round those values to the nearest whole number). Also, create a single graph that illustrates all three parts of this story. **(6 points)**

Part 1: The rat population increases $\boxed{\star\ y = 100(1.2)^x,\ x \le 2}$

Part 2: The rat population remains constant

when $x = 2$ $y = 100(1.2)^2 = 144$ so $\boxed{\star\ y = 144,\ 2 \le x \le 3}$

Part 3: The population decreases

$y =$ [redacted] $K(.5)^x$

when $x = 3$, $y = 144$ so $144 = K\left(\frac{1}{2}\right)^3$

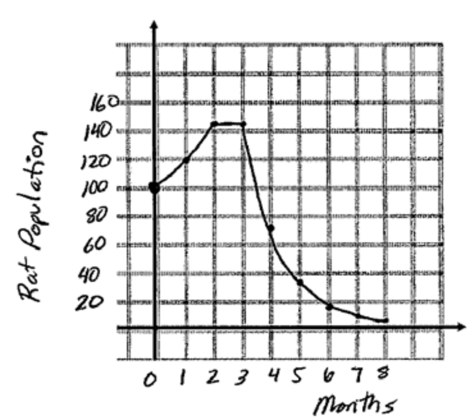
$8 \cdot 144 = K$

$1152 = K$

so $\boxed{\star\ y = 1152(.5)^x,\ x \ge 3}$

$y = 1152\left(\frac{1}{2}\right)^x$

x	y
4	72
5	36
6	18
7	9
8	4.5



Grade 8 Goal: Apply the Pythagorean Theorem

Green

A ladder is 15m long. Its foot is on a flat driveway 9m from the base of a vertical wall. How far up the wall will the top of the ladder reach?

Blue

The hypotenuse of a right angled triangle is twice the length of one leg of the triangle. The length of the other leg is 12 cm. How many square centimeters are in the area of the right triangle? Express your answer in simplest radical form.

Black

A swimmer at A, on one side of a straight-banked canal 100 meters wide, swims to a point B on the other bank, directly opposite to A. His steady rate of swimming is 5 meters per second, and the canal flow is a steady 3 meters per second.

A. Find the shortest time it could take him to swim from A to B.

B. Find the distance the swimmer would travel in this amount of time.

Blue Solution

12. The area of a rectangle is 60 square meters. The ratio of the rectangle's length to its width is **6:2.5**

What is the number of meters in the length of the rectangle's diagonal? (3 points)

Let x = the scale factor

$6x \cdot 2.5x = 60$

$15x^2 = 60$

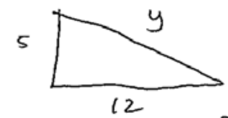
$x^2 = 4$

$x = 2$

So the side lengths are

$6x = 12$

$2.5x = 5$



$y^2 = 5^2 + 12^2$

$y^2 = 25 + 144 = 169 = y^2$

$y = \sqrt{169} = \boxed{13}$

Black Solution

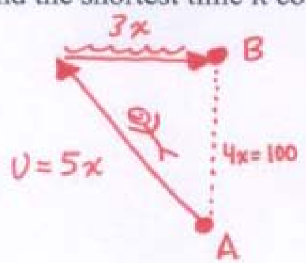
When $x = 1$, $\boxed{\text{AREA} = 15}$

9. A swimmer at A, on one side of a straight-banked canal 100 meters wide, swims to a point B on the other bank, directly opposite to A. His steady rate of swimming is 5 meters per second, and the canal flow is a steady 3 meters per second.

A. Find the shortest time it could take him to swim from A to B.

Let x = the # of seconds the swimmer swims

Let U = the distance he has travelled after swimming x seconds.



$(3x)^2 + (AB)^2 = (5x)^2$

$9x^2 + (AB)^2 = 25x^2$

$(AB)^2 = 16x^2$

$AB = 4x$

$4x = 100$

so $x = \boxed{25 \text{ seconds}}$

so $U = 5 \cdot 25 = \boxed{125 \text{ meters.}}$

B. Find the distance the swimmer would travel in this amount of time.